A Pathway to Equitable Math Instruction SEADTheme Guidebook: Identity

STRIDE



Identity SEAD Theme Guidebook Identity for Mathematics Grade 6–8

NCTM Definition of <u>Identity</u>: "the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives."

The Collaborative for Academic, Social and Emotional Learning (CASEL) defines the characteristics of Discourse:

- Self-Awareness: The ability to accurately recognize one's own emotions, thoughts, and values and how they influence behavior. The ability to accurately assess one's strengths and limitations, with a well grounded sense of confidence, optimism, and a "growth mindset."
- **Social Awareness**: The ability to **take the perspective of and empathize with others**, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports.
- **Responsible Decision-Making**: The ability to make constructive **choices about personal behavior and social interactions** based on ethical standards, safety concerns, and social norms. The realistic evaluation of consequences of various actions, and a consideration of the well-being of oneself and others.
- Self-Management: The ability to successfully regulate one's emotions, thoughts, and behaviors in different situations—effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals.
- **Relationship Skills**: The ability to **establish and maintain** healthy and rewarding relationships with diverse individuals and groups. The **ability to communicate** clearly, listen well, cooperate with others, resist inappropriate social pressure, negotiate conflict constructively, and seek and offer help when needed.

In addition, CASEL describes the characteristic of identity in mathematics as also including:

- Understand the links between personal and sociocultural identities that are defined by cultural and/or family values, ethnicity, race, socioeconomic status, gender, and other factors.
- Ground oneself in and affirm one's cultural heritage(s) or communities. (This can be especially
 important for students of color, and reduce <u>psychological distress</u> and <u>risky behaviors</u>, protect
 against the negative <u>health impacts</u> of racial discrimination, and promote a range of positive
 social and emotional outcomes, including <u>school engagement</u> and <u>prosocial behaviors</u>.)

Standards for Mathematical Practice¹ SMP 1: Make sense of problems and persevere in solving them.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students start by **explaining to themselves** the meaning of a problem and looking for entry points to its solution. They **analyze** givens, constraints, relationships, and goals. They **make conjectures** about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They **monitor and evaluate their progress** and change course if necessary. (Excerpted "Overview of Standards"².)

¹ "Overview of the Standards Chapters of the Mathematics Framework for California Schools," California Department of Education, 2015, <u>https://www.cde.ca.gov/ci/ma/cf/documents/mathfwoverview.pdf</u>.

² "Overview of the Standards Chapters"

Communicate that students' thinking is valued to build trust and rapport by asking questions that elicit students' thinking, such as when students are analyzing proportional relationships.

Examples referenced from Kansas Math Standards Flipbook:

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort—while procedures may be followed, the approach or pathway is not explicitly suggested by the task or task instructions, and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

Examples referenced from "A Framework for Re-envisioning Mathematics Instruction for ELLs":

- Support students in making sense of and solving problems rather than directly guiding them to answers.
- Allow sufficient time for students to productively struggle with learning in order to communicate the thinking behind their solutions to mathematical problems.

Examples referenced from "Supporting ELLs in Mathematics":

- **v** Materials consistently provide access to cognitively-demanding tasks.
- Teacher materials demonstrate when and how to support productive struggle before intervening.

| Student Action | Teacher Action |
|---|---|
| • Analyze information and explain the meaning of the problem. | Pose rich problems and ask open-ended questions. |
| • Actively engage in problem solving (develop, carry out, and refine a plan). | Provide wait-time for processing or finding solutions. |
| Show patience and positive attitudes. Ask themselves if their answers make | Circulate to pose probing questions and monitor student progress. |
| sense.Check their answers with a different | Provide opportunities and time for cooperative problem solving and reciprocal teaching. |
| method. | Interactions and discussions in cooperative problem solving as an EL support. |
| | Provide opportunities for EL students to respond to open ended questions. |

- Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Understand various approaches to solutions.
- **v** Supports for EL's as they maneuver through problem solutions and explore different pathways.
- **v** Providing dictionaries, vocabulary banks, translation apps to EL students.

This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice³ SMP 2: Reason abstractly and quantitatively.

(strong connection with Identity

Mathematically proficient students **make sense** of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. (Excerpted "Overview of Standards"⁴.)

³ "Overview of the Standards Chapters".

⁴ "Overview of the Standards Chapters"

| Student Action | Teacher Action |
|---|---|
| Represent a problem symbolically. | Ask students to explain their thinking regardless of accuracy. |
| • Explain their thinking. | Highlight flexible use of numbers. |
| Use numbers and quantities flexibly by applying properties of operations and place value. | Facilitate discussion through guided questions and representations. |
| • Examine the reasonableness of answers and calculations. | Accept varied solutions or representations. |

Example referenced from Student Achievement Partners document:

- Build a safe community where mathematical discourse supports active listening, promotes diverse perspectives and insights, and allows students to consider others' reasoning to advance their own mathematical understanding.
- For example, utilize a "which one doesn't belong?" activity for groups of students to discuss and analyze correspondences between graphs, tables, and equations that represent a relationship between dependent and independent variables.

Examples referenced from Capturing Quantities :

• Capturing Quantities is an instructional routine designed to focus students' attention on important quantities and relationships in problem situations.

Examples referenced from Kansas Math Standards Flipbook:

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.

Examples referenced from "Supporting ELLs in Mathematics":

Facilitate students' production of different kinds of reasoning and comparison of reasoning.

Examples referenced from **ELSF Mathematics Guidance**:

Materials demonstrate activities and ways to help students make meaning of typical mathematical texts such as word problems, graphs, tables, etc.

- Make sense of quantities and their relationships.
- **Decontextualize** (represent a situation symbolically and manipulate the symbols) and **contextualize** (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and be flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attend to the meaning of quantities, not just how to compute them.
- ▼ Support EL students in discussions and offer opportunities to ask questions.
- ▼ This icon is used to identify strategies and actions particularly beneficial to support ELLs in mathematics because they integrate language and content.

Standards for Mathematical Practice⁵ SMP 3: Construct viable arguments and critique the reasoning of others.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They **make conjectures** and build a logical progression of statements to explore the truth of their conjectures. They are able to **analyze** situations by breaking them into cases and can recognize and use counterexamples. They **justify** their conclusions, **communicate** them to others, and respond to the arguments of others. They **reason inductively** about data, making plausible arguments that take into account the context from which the data arose. (Excerpted "Overview of Standards"⁶.)

⁵ "Overview of the Standards Chapters"

 $^{^{\}rm 6}$ "Overview of the Standards Chapters"

Examples referenced from Kansas Math Standards Flipbook:

- **v** Structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

Examples referenced from <u>"Supporting ELLs in Mathematics</u>":

- **Recognize student's emerging** mathematical reasoning.
- Instruction should provide opportunities for students to actively engage in mathematical practices, such as reasoning, constructing arguments, and looking for and expressing structure and regularity.
- **Facilitate students' production** of different kinds of reasoning and comparison of reasoning.

Examples referenced from <u>"A Framework for Re-envisioning Mathematics Instruction for ELLs"</u>:

- Explain the reasoning behind correct answers, as well as the misconceptions behind incorrect responses, which enhances conceptual understanding of central math ideas.
- **v Develop** the student's confidence to communicate their mathematical understanding.
- Support students in refining their use of language to move toward more formal ways of describing, explaining, and justifying their reasoning in solving problems.

Examples referenced from ELSF Mathematics Guidance:

- Materials present opportunities for students to use language at different stages within a unit, such as speculating or predicting about a new topic, exploring and reflecting during an experience, presenting afterwards, etc.
- Activities deepen and extend learning through the various modes of communication: speaking, listening, reading, and writing.
- Materials include prompts for teachers to cultivate and facilitate back-and-forth mathematical discussions between students that refer to and build on each other's ideas.

| Student Action | Teacher Action |
|--|--|
| Make conjectures to explore their ideas. | Provide opportunities for students to listen to or read the conclusions and arguments of |
| Justify solutions and approaches. | others. |
| Listen to the reasoning of others, com- pare arguments, and decide whether the | Establish a safe environment for discussion. |
| arguments make sense. | Support ELs by use of L1 (primary language) and grouping to support L1. |
| Ask clarifying and probing questions. | • Ask clarifying and probing questions. |
| | Avoid giving too much assistance (e.g., providing answers or procedures). |

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.
- ▼ Support EL students in discussions and offer opportunities to ask questions.

[▼] This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice⁷ SMP 4: Model with mathematics.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students can **apply the mathematics they know** to solve problems arising **in everyday life, society, and the workplace**. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know **are comfortable making assumptions** and approximations to simplify a complicated situation, realizing that these may need revision later. (Excerpted "Overview of Standards"⁸.)

⁷ "Overview of the Standards Chapters"

 $^{^{\}rm 8}$ "Overview of the Standards Chapters"

Examples referenced from Kansas Math Standards Flipbook:

- ▼ Structures represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools), appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and **make it simpler by creating a model** that will represent the relationship between the quantities.

Examples referenced from <u>"Supporting ELLs in Mathematics</u>":

Instruction should provide opportunities for students to actively use mathematical language to communicate about mathematical situations.

Examples referenced from ELSF Mathematics Guidance:

- Materials highlight, define, illustrate, and show the purpose for mathematical language within the context of the lesson (not in isolation).
- Materials include prompts for students to reflect on their own thought processes, language use, methods, and learning of mathematical content.
- Teacher materials include relevant and practical suggestions for connecting mathematics content and practices to students' lives.
- **v** Materials encourage students to draw on prior knowledge, culture, and experiences.

| Student Action | Teacher Action |
|--|--|
| • Apply prior knowledge to new problems and reflect. | Pose problems connected to previous concepts. |
| Use representations to solve real-life problems. | Provide a variety of real-world contexts. |
| Apply formulas and equations where appropriate. Ask questions about the world around them and attempt to attach magningful. | Use intentional representations. Provide students the space to ask questions and pose problems about the world around them. |
| and attempt to attach meaningful mathematics to the world. | Support EL students in creating discussions, including asking and answering questions. |

- Understand reasoning quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the math they know to solve problems in everyday life.
- Simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram, and interpret the results of a mathematical situation.
- **Reflect** on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"
 - ▼ This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice⁹ SMP 5: Use appropriate tools strategically.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to **make sound decisions** about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. (Excerpted "Overview of Standards"¹⁰.)

⁹ "Overview of the Standards Chapters"

 $^{^{\}rm 10}$ "Overview of the Standards Chapters"

Example referenced from Student Achievement Partners document:

• Provide opportunities for students to consider tools they may use to solve a problem and justify their appropriateness. For example, they may choose to graph a function defined by expressions to picture the way one quantity depends on the other or use graphing technology to approximate solutions to a system of equations.

Examples referenced from Kansas Math Standards Flipbook:

- Lends itself to multiple learning tools. (Tools may include concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
 - a task when there is no need to have an exact answer.
 - a task when there is not enough information to get an exact answer.
 - a task to check if the answer from a calculation is reasonable.

Examples referenced from "A Framework for Re-envisioning Mathematics Instruction for ELLs":

v Provide tools to **enhance students' ability** to independently solve real-world problems.

| Student Action | Teacher Action |
|---|---|
| • Select and use tools strategically (and flexibly) to visualize, explore, and compare information. | Make appropriate tools available for learning (e.g., calculators, concrete models, digital resources, pencils and paper, compasses, protractors, etc.). |
| Use technological tools and resources to solve problems and deepen understanding. | Embed tools within instruction. |

- Use available tools, recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.
- Use mathematical models to visualize and analyze information.
 - ▼ This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice¹¹ SMP 6: Attend to precision.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students **try to communicate precisely** to others. They try to use clear definitions in discussion with others and **in their own reasoning**. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. (Excerpted "Overview of Standards"¹².)

¹¹ "Overview of the Standards Chapters"

 $^{^{\}rm 12}$ "Overview of the Standards Chapters"

Examples referenced from Kansas Math Standards Flipbook:

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

Examples referenced from "Supporting ELLs in Mathematics":

- Instruction should focus on uncovering, hearing, and supporting students' mathematical reasoning, not on the accuracy of their language.
- Focus on the mathematical meanings learners construct, not the mistakes they make or the obstacle they face.

Examples referenced from "A Framework for Re-envisioning Mathematics Instruction for ELLs":

 Provide learning opportunities for using formal mathematics vocabulary after students have had direct experience working on a math problem or concept, instead of pre-teaching vocabulary.

| Student Action | Teacher Action |
|---|--|
| Calculate accurately and efficiently. | Recognize and model efficient strategies for computation. |
| • Explain thinking using mathematics | |
| vocabulary. | Use mathematics vocabulary precisely and consistently, and challenge students to do |
| Use appropriate symbols and specify units of measure. | the same. |
| | Offer ELs supports and scaffold for language and vocabulary development and usage. |
| | |

- **Communicate precisely with others** and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.
 - ▼ This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice¹³ SMP 7: Look for and make use of structure.

(STRONG CONNECTION WITH IDENTITY)

Mathematically proficient students look **closely to discern** a pattern or structure. (Excerpted "Overview of Standards"¹⁴.)

Sample Actions / Instructional Strategies Related to Identity and SMP 7

- <u>Contemplate then Calculate</u> Contemplate then Calculate is an instructional routine designed to shift attention away from mindless calculations and toward necessary structural interpretations of mathematics. This routine fosters structural thinking.
- <u>Connecting Representations</u> is an instructional routine that positions students to think structurally, as they connect two representations, by articulating the underlying mathematics. An essential goal of this routine is expanding students' repertoire of structural noticings.

Examples referenced from Kansas Math Standards Flipbook:

- Requires students to look for the structure within mathematics in order to solve the problem (e.g., decomposing numbers by place value, working with properties, etc.).
- Asks students to take a complex idea and then identify and use the component parts to solve problems. (e.g., building on the structure of equal sharing, students connect the understanding to the traditional division algorithm). When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size."
- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. (e.g., 7 × 8 = (7 × 5) + (7 × 3) OR 7 × 8 = (7 × 4) + (7 × 4) new situations could be, distributive property, area of composite figures, multiplication fact strategies.)

Examples referenced from "A Framework for Re-envisioning Mathematics Instruction for ELLs":

v Use appropriate scaffolding to allow students to think about the mathematics they are learning.

¹³ "Overview of the Standards Chapters"

¹⁴ "Overview of the Standards Chapters"

| Student Action | Teacher Action |
|---|--|
| Look for, develop, and generalize relationships and patterns. | Provide time for applying and discussing properties. |
| • Apply conjectures about patterns and properties to new situations. | Ask questions about the application of patterns. |
| | Highlight different approaches for solving problems. |

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.
 - ▼ This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice¹⁵ SMP 8: Look for and express regularity in repeated reasoning.

(SOME CONNECTION WITH IDENTITY)

Mathematically proficient students **notice** if calculations are repeated and look both for general methods and for shortcuts. (Excerpted "Overview of Standards"¹⁶.)

Sample Actions / Instructional Strategies Related to Identity and SMP 8

Example referenced from Fostering Math Practices:

• <u>Recognizing Repetition</u> is an instructional routine that supports the difficult road to generalizing problem situations. Students enlist multiple modalities while they attend to the repetition in their counting, calculating, and constructing processes. In doing so, they leverage their repeated reasoning to make abstract generalizations.

Examples referenced from Kansas Math Standards Flipbook:

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

¹⁵ "Overview of the Standards Chapters"

 $^{^{\}rm 16}$ "Overview of the Standards Chapters"

| Student Action | Teacher Action |
|---|---|
| Look for methods and shortcuts through patterns in repeated calculations. | • Provide tasks and problems with patterns. |
| • Evaluate the reasonableness of intermediate results and solutions. | Ask about possible answers before computations are made, and inquire about reasonableness of answers after computations are made. |

Summary of Practice / Best Practices / Reflection for Identity and SMP8

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results.
 - This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Example of connections to priority mathematics content standards:

Students can develop **Identity** as they engage in a lesson related to "*Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers"* (7.NS.A). This can be accomplished by supporting students in SMP 4 utilizing the following actions and strategies.

- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Teacher materials include relevant and practical suggestions for connecting mathematics content and practices to students' lives.
- **v** Materials encourage students to draw on prior knowledge, culture, and experiences.

Glossary for Identity (from SMPs 1, 2, 3, 4, 5, 6, 7, 8)

Provide Access to understanding: to be able to use, enter, or get near (something) in order to understand it better

Analyze situations and problems: to study (something) closely and carefully; to learn the nature and relationship of the parts of (something) by a close and careful examination.

Apply math routines to similar or new problems: to put into operation or effect.

Ask useful questions to **clarify** or improve arguments/statements: to make (something) clear or clearer, such as to make (something) easier to understand.

Be aware in a cognitive situation: of, relating to, being, or involving conscious intellectual activity.

Communicate with others using a variety of methods: to convey knowledge of or information about.

Add conceptual thinking to a small group: of, relating to, or consisting of ideas, abstractions

Make conjectures: to offer an opinion or idea formed without proof or sufficient evidence.

Contextualize and decontextualize: to think about or provide information about the situation in which something happens (in its context); to decontextualize is to remove the context/situation in which something happens and to think about it more generally.

Elicit from oneself or from others: to call forth or draw out (something, such as information or a response).

Evaluate: to determine or fix the value of, or the appropriateness of

Examine claims: to look at (what someone says is true when some people may say it is not true) closely and carefully in order to learn more about it, to find problems, etc.

Make generalization: to summarize, to put together an overarching statement, law, principle, or proposition.

Justify one's thinking or solution: to prove or show.

Monitor: to watch, keep track of, or check usually for a special purpose.

Probe: to search into and explore very thoroughly.

Reasoning: the drawing of inferences or conclusions through the use of reason.

Recognize: to acknowledge or take notice of in some definite way.

Reflect: to express a thought or opinion about quality resulting from thinking about one's own work.

All definitions taken from the <u>Merriam Webster Learner's Dictionary (2020)</u>.